$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$ $E_b = \sigma T^4$

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$e^{i\pi} = -1$ $\frac{u}{u_\tau} = \frac{1}{\kappa}\ln\frac{yu_\tau}{\nu} + C$

USER GUIDE

Prandtl-Meyer Function Web Application

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Atkinson Science welcomes your comments on this User Guide. Please send an email to keith.atkinson@atkinsonscience.co.uk.

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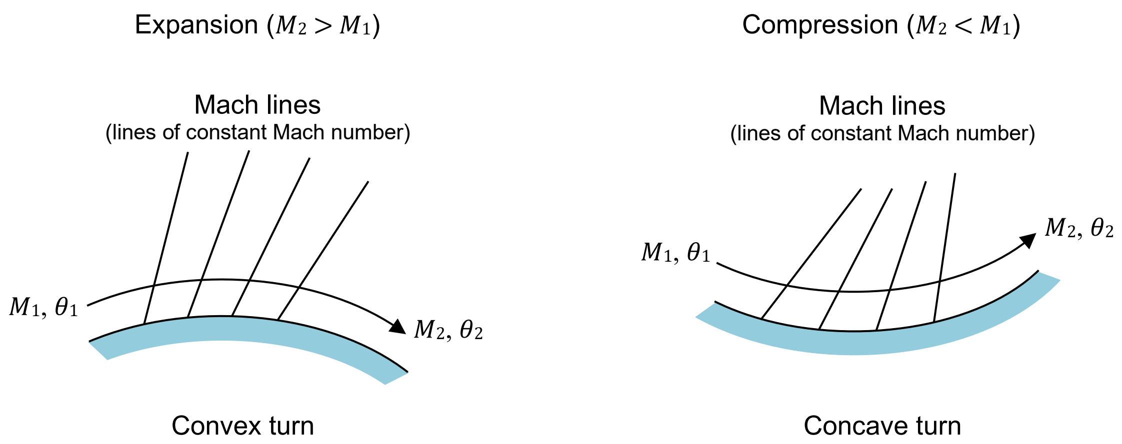
1 Introduction

You can find the Atkinson Science Prandtl-Meyer Function web application at the web address https://atkinsonscience.co.uk/WebApps/Aerospace/PrandtlMeyerFunction.aspx. The Prandtl-Meyer function is used to calculate the change in Mach number or flow inclination angle when a supersonic flow undergoes an isentropic expansion or compression by turning. Referring to Figure 1, an isentropic expansion by turning occurs when a gas flows over a convex corner, so that the flow along the wall is turned *away* from the main flow. An isentropic compression by turning occurs when a gas flows over a concave corner, so that the flow along the wall is turned *into* from the main flow. In Figure 1 the Mach number M and flow inclination angle θ change from (M_1, θ_1) to (M_2, θ_2). The Prandtl-Meyer function enables us to determine M_2 given M_1, θ_1 and θ_2, or θ_2 given M_1, θ_1 and M_2.

The web application was developed for use with the International Standard Atmosphere, Ref. [1], for which the ratio of specific heats γ is defined to be 1.4. Further information on the Prandtl-Meyer function can be found in Ref. [2], which can be downloaded from the Atkinson Science web site https://atkinsonscience.co.uk.

Figure 1 Isentropic expansion and compression by turning



2 User interface

The Prandtl-Meyer function has the form

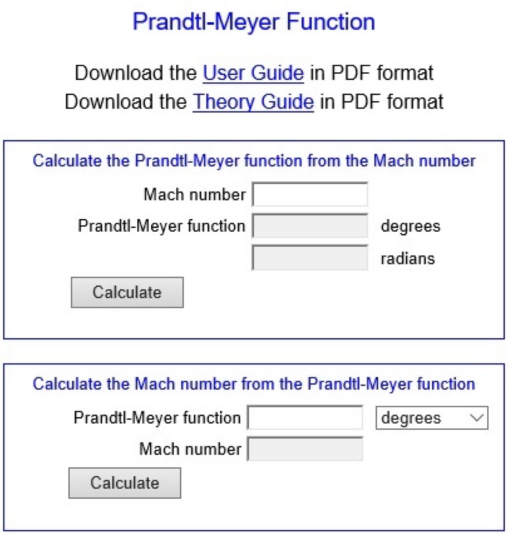
$$\theta = \nu(M)$$

The angle θ is zero when $M = 1$ and increases monotonically with M. The web application can calculate ν from M, or M from ν.

The user interface of the web application consists of two boxes, as shown in Figure 2. The user enters the Mach number M into the upper box and clicks the Calculate button in the box to obtain the Prandtl-Meyer function ν in degrees and in radians.

To obtain the Mach number M, the user enters the Prandtl-Meyer function ν in the lower box and clicks the Calculate button in the box. The user can choose the units in which ν is entered (degrees or radians) from the drop-down list.

Figure 2 User interface



3 Isentropic expansion

If the flow incidence angle θ_2 after an isentropic *expansion* is known, then the application can be used to calculate the Prandtl-Meyer function at exit from the corner ν_2:

$$\nu_2 = \nu_1 + |\theta_2 - \theta_1|$$

and thus the Mach number at exit M_2. Suppose a supersonic air flow has a Mach number $M_1 = 1.4$ and a flow incidence angle $\theta_1 = 15°$ on entering a convex corner and a flow incidence angle $\theta_2 = 23°$ on leaving. By entering $M_1 = 1.4$ into the upper box of the web application, we obtain $\nu_1 = 8.987°$. Then $\nu_2 = \nu_1 + |\theta_2 - \theta_1| = 8.987° + |23° - 15°| = 16.987°$. By entering $\nu_2 = 16.987°$ into the lower box of the web application, we obtain $M_2 = 1.672$.

If the exit Mach number M_2 is known, then the application can be used to calculate the exit flow incidence angle θ_2:

$$|\theta_2 - \theta_1| = \nu_2 - \nu_1$$

Suppose a supersonic air flow has a Mach number $M_1 = 1.8$ and a flow incidence angle $\theta_1 = 10°$ on entering a convex corner and a Mach number $M_2 = 2.5$ on leaving. By entering $M_1 = 1.8$ into the upper box of the web application, we obtain $\nu_1 = 20.725°$. Similarly, by entering $M_2 = 2.5$, we obtain $\nu_2 = 39.124°$. Then $\nu_2 - \nu_1 = 18.399°$. θ_2 can then be either 28.399° or −8.399°, but in both cases the flow is turned through 18.399°.

4 Isentropic compression

If the flow incidence angle θ_2 after an isentropic *compression* is known, then the application can be used to calculate the Prandtl-Meyer function at exit from the corner ν_2:

$$\nu_2 = \nu_1 - |\theta_2 - \theta_1|$$

and thus the Mach number at exit M_2. Suppose a supersonic air flow has a Mach number $M_1 = 2.1$ and a flow incidence angle $\theta_1 = 8°$ on entering a concave corner and a flow incidence angle $\theta_2 = 15°$ on leaving. By entering $M_1 = 2.1$ into the upper box of the web application, we obtain $\nu_1 = 29.097°$. Then $\nu_2 = \nu_1 - |\theta_2 - \theta_1| = 29.097° - |15° - 8°| = 22.097°$. By entering $\nu_2 = 22.097°$ into the lower box of the web application, we obtain $M_2 = 1.848$.

If the exit Mach number M_2 is known, then the application can be used to calculate the exit flow incidence angle θ_2:

$$|\theta_2 - \theta_1| = \nu_1 - \nu_2$$

Suppose a supersonic air flow has a Mach number $M_1 = 1.9$ and a flow incidence angle $\theta_1 = 18°$ on entering a concave corner and a Mach number $M_2 = 1.5$ on leaving. By entering $M_1 = 1.9$ into the upper box of the web application, we obtain $\nu_1 = 23.587°$. Similarly, by entering $M_2 = 1.5$, we obtain $\nu_2 = 11.905°$. Then $\nu_1 - \nu_2 = 11.682°$. θ_2 can then be either 29.682° or 6.318°, but in both cases the flow is turned through 11.682°.

5 Ranges of parameters

In the web application γ is 1.4, which is the ratio of specific heats for the International Standard Atmosphere (Ref. [1]). When calculating the Prandtl-Meyer function from the Mach number, the Mach number may not be less than 1. When calculating the Mach number from the Prandtl-Meyer function, the Prandtl-Meyer function may not be less than 0° or greater than 130° (2.27 radians).

6 References

1. *International Standard Atmosphere*, ISO 2533:1975, International Standards Organisation, 1975.
2. K. N. Atkinson, *Prandtl-Meyer Function*, *Theory Guide*, Atkinson Science Limited, 22 September 2020 (download from https://atkinsonscience.co.uk).